

Dynamical Feedback of Self-generated Magnetic Fields in Cosmic Ray Modified Shocks

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ABSTRACT

We present a semi-analytical kinetic calculation of the process of non-linear diffusive shock acceleration (NLDSA) which includes the magnetic field amplification due to cosmic ray induced streaming instability, the dynamical reaction of the amplified magnetic field and the possible effects of turbulent heating. The approach is specialized to parallel shock waves and the parameters we chose are the ones appropriate to forward shocks in Supernova Remnants. Our calculation allows us to show that the net effect of the amplified magnetic field is to enhance the maximum momentum of accelerated particles while reducing the concavity of the spectra, with respect to the standard predictions of NLDSA. This is mainly due to the dynamical reaction of the amplified field on the shock, which noticeably reduces the modification of the shock precursor. The total compression factors which are obtained for parameters typical of supernova remnants are $R_{tot} \sim 7 - 10$, in good agreement with the values inferred from observations. The strength of the magnetic field produced through excitation of streaming instability is found in good agreement with the values inferred for several remnants if the thickness of the X-ray rims are interpreted as due to severe synchrotron losses of high energy electrons. We also discuss the relative role of turbulent heating and magnetic dynamical reaction in driving the reduction of the precursor modification.

Key words: acceleration of particles - SNR - shock waves - magnetic field amplification - jump conditions

1 INTRODUCTION

The supernova remnant (SNR) paradigm for the origin of galactic cosmic rays heavily relies on the mechanism for particle acceleration being the *Diffusive Shock Acceleration* (DSA) at the shock front generated in the supernova blast. This mechanism, also known as *first order Fermi acceleration*, has been studied in great detail and a two decades long work has led to the development of a kinetic non-linear theory that allows us to assess the importance of the dynamical reaction of the accelerated particles on the shock itself. The nonlinear effects described by the theory turn out to be not just corrections. They rather reveal the profound reasons why the mechanism works in the first place as a cosmic ray accelerator: the large efficiency required to explain the energetics (10 – 20% of the kinetic energy of the blast wave going into cosmic rays) and the large magnetic fields required to explain the maximum energies observed in cosmic rays are the two main motivations for developing such a nonlinear theory, and, not surprisingly, are among the most successful predictions of the theory.

The initial attempt at building a nonlinear theory led to two-fluid models (Drury & Völk 1980, 1981) which provided information on the dynamics and thermodynamics of the shocked gas and cosmic ray gas but not on the spectrum of the accelerated particles. A satisfactory kinetic approach, able to predict the spectrum of accelerated particles was later proposed by Malkov (1997); Malkov, Diamond & Völk (2000) and Blasi (2002, 2004). More recently a kinetic model which takes into account the possibility of arbitrary diffusion coefficients was put forward by Amato & Blasi (2005). The self-excitation of unstable modes leading to magnetic field amplification was then also introduced by Amato & Blasi (2006).

Parallel to these analytical approaches, developed primarily in the assumption of quasi-stationarity of the acceleration process, numerical approaches following the temporal evolution were also developed (Bell 1987; Jones & Ellison 1991; Ellison, Möbius & Paschmann 1990; Ellison, Baring & Jones 1995, 1996; Kang & Jones 1997, 2005; Kang, Jones & Gieseler 2002). These have been of crucial importance for the description of some aspects of the phenomenology connected with the acceleration process in SNRs, especially during the Sedov-Taylor phase. Numerical methods for the solution of the transport equation for cosmic rays and of the conservation equations for the plasma in the shock region have been exten-

Table 1. Parameters for 5 well studied SNRs.

SNR	u_0 (km/s)	B_2 (μ G)	$\alpha_2 \times 10^3$
Cas A	5200 (2500)	250–390	32 (36)
Kepler	5400 (4500)	210–340	23 (25)
Tycho	4600 (3100)	300–530	27 (31)
SN 1006	2900 (3200)	91–110	40 (42)
RCW 86	(800)	75–145	14–35 (16–42)

The value are from Völk, Berezhko & Ksenofontov (2005) (in parentheses) and from Parizot et al. (2006). In order to estimate the normalized downstream magnetic pressure $\alpha_2 = B_2^2/(8\pi\rho_0 u_0^2)$, for the SNRs discussed by Parizot et al. (2006) we used $\rho_0 = 0.1 m_p/\text{cm}^3$ (SN 1006) and $\rho_0 = 0.5 m_p/\text{cm}^3$ (in the other cases). Völk, Berezhko & Ksenofontov (2005), instead, provide directly α_2 .

sively used to reproduce the observed multifrequency observations from single SNRs (e.g. Völk, Berezhko & Ksenofontov 2005; Cassam-Chenaï et al. 2005).

A typical result of all approaches to nonlinear shock acceleration is that the spectra of accelerated particles are far from being power laws. The concave shape leads to spectra as flat as $p^{-3.2}$ close to the maximum momentum, corresponding to total compression factors that may exceed $\sim 50 - 100$ (Amato & Blasi 2005). In general the total compression factor R_{tot} is found to scale with the Mach number as $R_{tot} \propto M_0^{3/4}$ (Berezhko & Ellison 1999). Such levels of shock modification do not compare well with some observations, which suggest $R_{tot} \sim 7 - 10$ (see e.g. Völk, Berezhko & Ksenofontov 2005).

From the phenomenological point of view this problem has been faced by invoking some sort of turbulent heating: part (or all) of the energy in the form of Alfvén waves which are responsible for the scattering of charged particles is assumed to be damped on the thermal gas, thereby causing its heating in the upstream precursor region (Völk & McKenzie 1981; McKenzie & Völk 1982). This process, originally investigated as a possible mechanism to limit the magnetic field amplification, keeping the turbulent field in the linear regime (e.g. $\delta B/B \ll 1$), is currently called upon also in situations in which $\delta B/B \gg 1$.

An important piece of information has been recently added to the debate on whether SNRs can be the sources of galactic cosmic rays: Chandra X-ray observations of some remnants showed narrow filaments of non-thermal origin (see Vink 2006, and references therein for a review). If the thickness of the rims is assumed to be due to severe synchrotron losses limiting the lifetime of high energy electrons, then one can infer the strength of the magnetic field in the downstream region, which turns out to be ~ 100 times stronger than magnetic fields in the ISM.

On the other hand, it has been proposed that the narrow rims may reflect the damping scale of the magnetic turbulence rather than the loss length of electrons (Pohl et al. 2005).

This interpretation is at odds with the morphology of the radio emission, as discussed by Rothenflug et al. (2004) (see also the discussion in Morlino et al. 2009), but at present it is not possible to rule it out. If it turns out to be correct, then there would be no observational constraint on the magnetic field in the shock region.

If on the other hand the interpretation based on strong synchrotron losses is confirmed, the inferred levels of magnetization can be interpreted as a result of streaming instability induced by cosmic rays efficiently accelerated at a parallel shock front (Bell 1978; Bell & Lucek 2001; Bell 2004), although alternative mechanisms have also been put forward (e.g. Giacalone & Jokipii 2007). In Table 1 we list the SNRs where evidence has been collected, from X-ray observations, for a strong magnetic field: u_0 is the shock velocity, B_2 is the value of the magnetic field downstream of the shock as inferred from the X-ray brightness profile and α_2 is the magnetic energy density immediately downstream of the forward shock, in units of the total kinetic pressure at upstream infinity ($\alpha_2 = B_2^2/(8\pi\rho_0 u_0^2)$). The data are from Völk, Berezhko & Ksenofontov (2005) (numbers in parentheses) and from Parizot et al. (2006).

The efficient acceleration and the magnetic field amplification are the two most impressive manifestations of nonlinear diffusive acceleration at SNR shocks. Both these aspects have been included in a Monte Carlo scheme by Vladimirov, Ellison & Bykov (2006), with the magnetic field amplification described accordingly to the phenomenological scenario proposed by Bell & Lucek (2001). Vladimirov, Ellison & Bykov (2006) find that, when amplification is efficient, the wave pressure makes the plasma upstream of the shock less compressible and the change in the energy density of the magnetic turbulence across the subshock strongly affects R_{sub} , which in turn affects the injection efficiency and the entire process of cosmic ray acceleration.

In a previous paper (Caprioli et al. 2008a) we used a three-fluid approach (gas, cosmic rays and Alfvén waves), to show the very general nature of the magnetic dynamical feedback: the shock dynamics is significantly affected by the turbulence backreaction as soon as the magnetic pressure becomes comparable to that of the gas upstream of the subshock. In particular, for the magnetization levels inferred from available observations (see Tab. 1) and the saturation values that are expected from different amplification mechanisms proposed in the literature, we found that R_{tot} would naturally lead to values in the range 6-10, in good agreement with the values currently inferred from observations.

In the present paper we use a kinetic model for particle acceleration at a parallel shock

in the non-linear regime, together with the modified conservation equations in the precursor and at the subshock in order to describe particle acceleration, the dynamical reaction of accelerated particles, the generation of magnetic field through streaming instability and the dynamical reaction of the magnetic field on the shock, which also results in modified cosmic ray spectra.

The paper is structured as follows: in Sec. 2 we write down the correct jump conditions for a parallel, modified shock when magnetic turbulence is excited by the cosmic ray streaming, as in Caprioli et al. (2008a). In Sec. 3 we summarize our kinetic model based on that by Amato & Blasi (2006), including the self-consistent treatment of the magnetic field amplification via resonant streaming instability. The latter is described in detail in Sec. 4. In Sec. 5 we present our main results, namely the solutions for DSA with resonant streaming instability: in particular we discuss the reduced modification of the precursor and the consequent steepening of the spectrum near the (increased) maximum momentum. In Sec. 6 we investigate the combined effect of magnetic reaction and turbulent heating and we show that the dominant effect on the precursor modification is likely to be that of the magnetic reaction. Our conclusions are in Sec. 7.

2 DYNAMICS OF A MAGNETIZED COSMIC RAY MODIFIED SHOCK

The pressure of the accelerated particles upstream of the shock surface leads to the formation of a shock precursor, in which the fluid speed gradually decreases while approaching the shock. One can describe this effect by introducing two compression factors $R_{tot} = u_0/u_2$ and $R_{sub} = u_1/u_2$, where u is the fluid velocity and the indexes '0', '1' and '2' refer, here and in the following, to quantities taken at upstream infinity ($x = -\infty$), and immediately upstream ($x = 0^-$) and downstream ($x = 0^+$) of the subshock, respectively.

We consider a non-relativistic plane shock whose normal is parallel to the background magnetic field B_0 . The equations defining the jump conditions at the shock surface in the presence of cosmic rays and self-generated Alfvén waves can be written as

$$[\rho u]_1^2 = 0, \quad (1)$$

$$[\rho u^2 + p + p_w]_1^2 = 0, \quad (2)$$

$$\left[\frac{1}{2} \rho u^3 + \frac{\gamma}{\gamma - 1} u p + F_w \right]_1^2 = 0, \quad (3)$$

where ρ , u , p and γ stand for density, velocity, pressure and ratio of specific heats of the

gas, p_w and F_w are the magnetic pressure and energy flux and the brackets indicate the difference between quantities downstream and upstream of the subshock ($[X]_1^2 = X_2 - X_1$). Some subtleties of the treatment of mass, momentum and energy conservation in cosmic ray modified shocks are extensively discussed in Caprioli, Blasi, Amato (2008b). Here we only want to emphasize that the contribution of the terms related to the pressure and energy flux of the cosmic rays (p_c and F_c) disappears when considering the subshock (i.e. p_c and F_c are both continuous across the subshock). Also the contribution to the pressure by the background magnetic field vanishes, \vec{B}_0 being parallel to \vec{u} and therefore unaffected by any fluid compression. The subshock Rankine-Hugoniot relations are therefore not affected by the contribution of cosmic rays, but they do take into account the presence of magnetic turbulence, leading to a magnetized gaseous shock.

In order to infer the magnetic field jump conditions, we use the approach of Scholer & Belcher (1971) and Vainio & Schlickeiser (1999) to describe the transmission and reflection coefficients appropriate for Alfvén waves at the shock surface. Following Vainio & Schlickeiser (1999), we account for two upstream wave trains with helicities $H_c = \pm 1$, and for their respective counterparts downstream. It is worth stressing that, in principle, this part of the problem can be a complex one, because waves can be transmitted and reflected also within the precursor, due to the gradient in all quantities there. In general this could lead to isotropization of the turbulence and to the generation of a wave train propagating in the direction opposite to that directly excited by the cosmic ray streaming (i.e. the one with $H_c = -1$). However, we do not expect this effect to be very relevant because the shocks we are dealing with are not very modified, as we can check *a posteriori*. For these reasons we neglect turbulence isotropization in the precursor, but we include the transmission and reflection of Alfvén waves at the subshock surface, which enter the jump conditions.

Let $\delta\vec{B}_\mu$ be a given mode, indicated by the subscript μ , of the magnetic turbulence. We write the corresponding velocity perturbation simply as

$$\delta\vec{u}_\mu = -H_{c,\mu} \frac{\delta\vec{B}_\mu}{\sqrt{4\pi\rho}}. \quad (4)$$

Neglecting the electric field contribution, which is of order u^2/c^2 , the magnetic pressure and the energy flux are respectively

$$p_w = \frac{1}{8\pi} \left(\sum_\mu \delta\vec{B}_\mu \right)^2, \quad (5)$$

$$F_w = \sum_{\mu} \frac{\delta \vec{B}_{\mu}^2}{4\pi} (u + H_{c,\mu} v_A) + \frac{\left(\sum_{\mu} \delta \vec{B}_{\mu}\right)^2}{8\pi} u, \quad (6)$$

where the sum over μ has to be intended as the sum over all the wave modes present at a given position.

In obtaining these relations we explicitly used the fact that, for Alfvén waves, F_w in Eq. 6 has two contributions: the former is the normal component of the Poynting vector $(\vec{B} \times \delta \vec{u} + \delta \vec{B} \times \vec{u}) \times \delta \vec{B} / 4\pi$, while the latter represents the kinetic energy flux in transverse velocity, $\rho/2 \delta \vec{u}^2 u$ (with δu given by Eq. 4). If the turbulence manifests itself in forms other than resonant Alfvén waves, p_w and F_w may differ significantly. We allow the generated waves to have both helicities, hence the sum over the modes should account, in the upstream region, for both forward- and backward-going waves, namely $\delta \vec{B}_{1\pm}$. Each of these modes, however, when advected behind the subshock has to split into two waves with opposite helicities in order to satisfy the Maxwell equations at the subshock (see e.g McKenzie & Westphal 1969; Scholer & Belcher 1971). This fact leads to four different downstream modes, which can be described by the introduction of proper transmission and reflection coefficients. In this scenario a “reflected” (“transmitted”) wave is a wave with a helicity opposite (equal) to the one of its upstream counterpart. It is worth stressing that, the fluid being super-Alfvénic either upstream or downstream, all the modes are actually advected with the fluid, independent of their direction of propagation.

These coefficients, \mathcal{T} and \mathcal{R} respectively, were derived by McKenzie & Westphal (1969), and for each upstream helicity $H_c = \pm 1$ read:

$$\mathcal{T} = \frac{R_{sub} + \sqrt{R_{sub}}}{2} \frac{M_{A1} + H_c}{M_{A1} + \sqrt{R_{sub}} H_c} \simeq \frac{R_{sub} + \sqrt{R_{sub}}}{2}, \quad (7)$$

$$\mathcal{R} = \frac{R_{sub} - \sqrt{R_{sub}}}{2} \frac{M_{A1} + H_c}{M_{A1} - \sqrt{R_{sub}} H_c} \simeq \frac{R_{sub} - \sqrt{R_{sub}}}{2}. \quad (8)$$

Here $M_A = u/v_A$ is the Alfvénic Mach number, namely the ratio between fluid and Alfvén speed, which is of order 100 and more for a typical supernova shock. For each sign of H_c we have $\delta B_2 / \delta B_1 = \mathcal{T} + \mathcal{R} = R_{sub}$, and hence

$$p_{w2} = p_{w1} R_{sub}^2. \quad (9)$$

Since the subshock can be viewed as a magnetized gas shock, as we already stressed, the jump conditions found by Vainio & Schlickeiser (1999) for the pressure and temperature hold also in our case and can be written respectively as:

$$\frac{p_2}{p_1} = \frac{(\gamma + 1)R_{sub} - (\gamma - 1) + (\gamma - 1)(R_{sub} - 1)\Delta}{\gamma + 1 - (\gamma - 1)R_{sub}}, \quad (10)$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \frac{1}{R_{sub}}, \quad (11)$$

with Δ defined as:

$$\Delta = \frac{R_{sub} + 1}{R_{sub} - 1} \frac{[p_w]_1^2}{p_1} - \frac{2R_{sub}}{R_{sub} - 1} \frac{[F_w]_1^2}{p_1 u_1}. \quad (12)$$

Using the expressions in Eqs. 7 and 8 for the transmitted and reflected Alfvén waves, we find:

$$\Delta = (R_{sub} - 1)^2 \frac{p_{w1}}{p_1} + R_{sub} \frac{\delta \vec{B}_{1-} \cdot \delta \vec{B}_{1+}}{2\pi p_1}. \quad (13)$$

Following Scholer & Belcher (1971) and Vainio & Schlickeiser (1999), we assume that the two opposite-propagating waves carry magnetic fields $\delta \vec{B}_{1\pm}$ displaced in such a way that $\delta \vec{B}_{1-} \cdot \delta \vec{B}_{1+} = 0$. This is not the most general possible configuration, but still it is indeed expected to be the most common, since it describes situations in which there is only one wave train, or the two fields are reciprocally orthogonal, or even, on average, when the relative phase between the wave trains is arbitrary.

Hereafter we use quantities normalized to the values of ram-pressure and velocity at upstream infinity:

$$U(x) = \frac{u(x)}{u_0} \quad \alpha(x) = \frac{(\sum_{\mu} \delta \vec{B}_{\mu})^2}{8\pi \rho_0 u_0^2} \quad P(x) = \frac{p(x)}{\rho_0 u_0^2} \quad \xi(x) = \frac{p_c(x)}{\rho_0 u_0^2}. \quad (14)$$

If the heating of the upstream plasma is due only to adiabatic compression, using the mass conservation $\rho(x)u(x) = \rho_0 u_0$, the normalized plasma pressure can be written as

$$P(x) = \frac{U(x)^{-\gamma}}{\gamma M_0^2}, \quad (15)$$

where as usual M_0 is the sonic Mach number at upstream infinity.

Substituting Eq. 10, Eq. 13 and the above expression for $P(x)$ in the equation for momentum conservation, the compression factors R_{sub} and R_{tot} are related through the equation

$$R_{tot}^{\gamma+1} = \frac{M_0^2 R_{sub}^{\gamma}}{2} \left[\frac{\gamma + 1 - R_{sub}(\gamma - 1)}{1 + \Lambda_B} \right], \quad (16)$$

which is the same as the standard relation (see e.g. Blasi 2004) apart from the factor $(1 + \Lambda_B)$, where

$$\Lambda_B = W [1 + R_{sub} (2/\gamma - 1)] , \quad (17)$$

and we have defined

$$W = \alpha_1 / P_1. \quad (18)$$

It is clear that the net effect of the magnetic turbulence is to make the fluid less compressible: if $W \gtrsim 1$, R_{tot} may be considerably reduced, while the pressure (and temperature) jump increases, according to Eq. 10 (and 11). In Caprioli et al. (2008a) we showed, by means of purely hydrodynamical considerations, and without any assumptions on the details of particle acceleration and magnetic field generation, that this is very likely the case for the SNRs listed in Tab. 1.

As a final remark, we notice that if one naively assumed that downstream $F_{w2} = 3u_2p_{w2}$, instead of using the appropriate transmission and reflection coefficients (which come from the need of satisfying Maxwell equations at the subshock), one would obtain

$$\Delta' = [(R_{sub} - 1)^2 - 2R_{sub}]W < \Delta. \quad (19)$$

Using Δ' rather than Δ leads to an incorrect estimate of the pressure jump (Eq. 10), which in this case may even turn out to be smaller than for an unmagnetized shock ($\Delta' < 0$ for $R_{sub} < 3.73$). At the same time, one would have $\Lambda'_B = W[1 + R_{sub}(3/\gamma - 2)]$ in Eq. 16 and hence find a less marked decrease of R_{tot} .

3 THE KINETIC SELF-CONSISTENT SOLUTION

In this section we describe the calculations that lead to an exact solution for the spectrum and the spatial distribution of the particles accelerated at a non-linear astrophysical shock, including the generation of Alfvén waves by the same particles and the dynamical reaction of both cosmic rays and magnetic turbulence on the fluid. This method is based on the kinetic treatment of the problem in the stationary regime proposed by Amato & Blasi (2005, 2006), which also allows for an arbitrary choice of spatial and momentum dependence of the diffusion coefficient $D(x, p)$. In the following we consider Bohm diffusion in the self-generated magnetic field, i.e. we set

$$D(x, p) = \frac{1}{3}cr_L(\delta B) = \frac{1}{3}c\frac{pc}{eB(x)}, \quad (20)$$

where r_L is the the Larmor radius of a particle of momentum p in the local amplified magnetic field $B(x) = \sqrt{8\pi\alpha(x)\rho_0u_0^2}$. Needless to say that this form of the diffusion coefficient is basically only an ansatz and whether Nature provides such a diffusion coefficient is at present not clear.

The transport of accelerated particles consists of both advection and diffusion. The correct treatment of these processes in the limit of small perturbation amplitudes ($\delta B \ll B_0$),

also including the presence of both wave trains, are well known and can be found e.g. in the work by Skilling (1975a,b). Unfortunately, in the case of strong magnetic field amplification a full theory of cosmic ray transport is still missing, and even the definition of an effective wave velocity, v_A , is troublesome. A semi-analytical treatment, which is what we are interested in, is only possible within the framework of quasi-linear theory. We adopt an Alfvén velocity defined as

$$v_A(x) = \frac{B_0}{\sqrt{4\pi\rho(x)}}, \quad (21)$$

where $\rho(x)$ is the gas density in the precursor at the position x and B_0 is the strength of the unperturbed magnetic field. In the unlikely case that the waves could keep their Alfvénic nature even in the strong turbulence regime, this wave velocity would in fact be well defined.

As to the interactions between streaming particles and Alfvén waves, we do not neglect the velocity of the scattering centers v_A with respect to the fluid velocity $u(x)$. This means that, in principle, the compression ratios of the background fluid are different from the ones experienced by the scattering centers, which turn out to be

$$S_{sub} = \frac{u_1 - v_{A1}}{u_2 + v_{A2}} \quad S_{tot} = \frac{u_0 - v_{A0}}{u_2 + v_{A2}} \simeq \frac{u_0}{u_2 + v_{A2}}. \quad (22)$$

In the recipes above we have implicitly assumed that upstream waves are produced preferentially with $H_c = -1$. Behind the subshock, instead, the net velocity of the two opposite-propagating wave trains, when calculated in the downstream gas reference frame (v_{A2}), turns out to be in the same direction as the fluid one (see also Bell 1978).

In a typical SNR the condition $v_A \ll u$ usually holds, hence the difference between (S_{sub}, S_{tot}) and (R_{sub}, R_{tot}) is expected not to be very relevant. But if for some reason M_A is small enough, the compression ratios felt by the accelerated particles may be significantly different with respect to the fluid ones, leading to a modified spectral slope, as already showed by Bell (1978). This is why we retain v_A in the calculations and check *a posteriori* that in the cases considered this correction is not important. In Sec. 5.1 we investigate the consequences of adopting a different prescription for the velocity of the scattering centers.

From the kinetic point of view, cosmic rays are described by their distribution function in phase space $f(\vec{x}, \vec{p})$. Keeping only the isotropic part (since $f(\vec{p}) = f(p) + O(u^2/c^2)$) and recalling that the shock is non-relativistic, the diffusion-advection equation for a one-dimensional shock reads:

$$[u(x) - v_A(x)] \frac{\partial f(x, p)}{\partial x} = \frac{\partial}{\partial x} \left[D(x, p) \frac{\partial}{\partial x} f(x, p) \right] + \frac{d[u(x) - v_A(x)]}{dx} \frac{p}{3} \frac{\partial f(x, p)}{\partial p} + Q(x, p), \quad (23)$$

with $Q(x, p)$ the injection of particles in the accelerator. We assume that injection occurs only at the shock location ($x = 0$) and at momentum p_{inj} , involving a fraction η of the particles crossing the shock, such that

$$Q(x, p) = \eta \frac{\rho_1 u_1}{4\pi m_p p_{inj}^2} \delta(p - p_{inj}) \delta(x). \quad (24)$$

For the fraction η of injected particles we adopt the recipe of Blasi, Gabici & Vannoni (2005):

$$\eta = \frac{4}{3\sqrt{\pi}} (S_{sub} - 1) \psi^3 e^{-\psi^2}, \quad (25)$$

which assumes that only particles with momentum $p_{inj} \geq \psi p_{th,2}$ (i.e. ψ times the thermal particles' momentum downstream) can be accelerated. This recipe fits well within our self-consistent approach because it only involves $p_{th,2}$ and S_{sub} , which are both outputs of the calculation, rather than free parameters. Moreover, it self-limits the acceleration process, suppressing the injection when particle acceleration is too efficient, in which case the large shock modification leads to $S_{sub} \rightarrow 1$ and $\eta \rightarrow 0$. In the following, we consider values of ψ between 3.5 and 4, corresponding to η between $\sim 10^{-4}$ and $\sim 10^{-5}$.

As shown by Amato & Blasi (2005, 2006) and then by Blasi, Amato & Caprioli (2007), a very good approximation for the solution of Eq. 23, $f(x, p)$, is found in the form:

$$f(x, p) = f_1(p) \exp \left[\frac{S_{sub} - 1}{S_{sub}} \frac{q(p) u_0}{3} \int_0^x dx' \frac{U(x') - V_A(x')}{D(x', p)} \right], \quad (26)$$

where $V_A = v_A/u_0$, $f_1 = f(0, p)$ and $q(p) = -\frac{d \log f_1(p)}{d \log p}$ is the spectral slope at the shock location. The above expression reduces to the correct distribution function in the test particle limit and exactly satisfies the jump conditions at the subshock, as obtained by integrating Eq. 23 from 0^- to 0^+ .

As shown by Blasi (2002), $f_1(p)$ can be written as

$$f_1(p) = \left(\frac{3S_{tot}}{S_{tot}U_p(p) - 1} \right) \frac{\eta \rho_1}{4\pi m_p p_{inj}^3} \exp \left[- \int_{p_{inj}}^p \frac{dp'}{p'} \frac{3S_{tot}U_p(p')}{S_{tot}U_p(p') - 1} \right], \quad (27)$$

where we defined

$$U_p(p) = U_1 - V_{A1} - \frac{1}{f_1(p)} \int_{-\infty}^0 dx f(x, p) \frac{d[U(x) - V_A(x)]}{dx}. \quad (28)$$

The normalized pressure in cosmic rays is written in terms of this solution as

$$\xi(x) = \frac{4\pi}{3\rho_0 u_0^2} \int_{p_{inj}}^{p_{max}} dp p^3 v(p) f_1(p) \exp \left[\int_0^x dx' \frac{U(x') - V_A(x')}{x_p(x', p)} \right], \quad (29)$$

having defined

$$x_p(x, p) = \frac{3S_{sub}}{S_{sub} - 1} \frac{D(x, p)}{u_0 q(p)}, \quad (30)$$

and p_{max} as the maximum momentum achievable by the accelerated particles. We determine the latter self-consistently, using the calculations by Blasi, Amato & Caprioli (2007) for cosmic ray modified shocks, assuming that the particles' maximum energy is limited by the acceleration time rather than by the size of the system. The time needed to accelerate particles up to p_{max} turns out to be

$$\tau(p_{max}) = \frac{3R_{tot}}{u_0^2} \int_{p_{inj}}^{p_{max}} \frac{dp'}{p'} \frac{1}{R_{tot}U_p(p') - 1} \left[R_{tot}D_2(p') + \frac{u_0}{f_1(p')} \int_{-\infty}^0 dx f(x, p') \right]. \quad (31)$$

Integrating by parts Eq. 28 it is possible to express $U_p(p)$ in terms of $U(x)$ and $x_p(x, p)$ alone:

$$U_p(p) = \int_{-\infty}^0 dx \frac{[U(x) - V_A(x)]^2}{x_p(x, p)} \exp \left[\int_0^x dx' \frac{U(x') - V_A(x')}{x_p(x', p)} \right]. \quad (32)$$

Then, differentiating Eq. 29 with respect to x , we obtain

$$\frac{d\xi(x)}{dx} = \xi(x)\lambda(x) [U(x) - V_A(x)], \quad (33)$$

where

$$\lambda(x) = \frac{\int_{p_{inj}}^{p_{max}} dp p^3 v(p) \frac{1}{x_p(x, p)} f(x, p)}{\int_{p_{inj}}^{p_{max}} dp p^3 v(p) f(x, p)}. \quad (34)$$

Finally, we need a relation which describes how the Alfvén waves are excited by the streaming cosmic rays and how the wave energy is transported in the precursor. Very generally, we can assume $\alpha(x)$ to be a function of $\xi(x)$ and $U(x)$. Using the equation of momentum conservation between a point x in the precursor and upstream infinity, it is therefore possible to write $\alpha(x)$ as a function of $U(x)$ only (see e.g. Eq. 42 in the next section, where a discussion of magnetic field amplification due to resonant streaming instability will be presented.)

The nonlinear system defined by Eqs. 23-34 can be solved, for a given age of the system, with three nested iterations. We guess a value for p_{max} , as the starting point of the outermost cycle. Then, we fix a value for the ratio R_{sub}/R_{tot} , and derive the corresponding R_{sub} and R_{tot} from Eq. 16. The equation for conservation of momentum,

$$U(x) + \xi(x) + \alpha(x) + P(x) = 1 + \frac{1}{\gamma M_0^2}, \quad (35)$$

once it is evaluated in 0^- ,

$$\frac{R_{sub}}{R_{tot}} + \frac{1}{\gamma M_0^2} \left(\frac{R_{tot}}{R_{sub}} \right)^\gamma + \xi_1 + \alpha_1 = 1 + \frac{1}{\gamma M_0^2}, \quad (36)$$

only involves ξ_1 and R_{sub}/R_{tot} , since α_1 can be written as a function of R_{sub}/R_{tot} , as it will be shown in the next section (Eq. 42). Eq. 36 gives the boundary condition ξ_1 for Eq. 33.

The latter is then solved recursively: the solution at step n is calculated using $U(x)$ and $\lambda(x)$ at step $n - 1$:

$$\xi^{(n)}(x) = \xi_1 \exp \left[\int_0^x dx' \lambda^{(n-1)}(x') [U(x') - V_A(x')]^{(n-1)} \right]. \quad (37)$$

The iteration on n is carried out until convergence is reached. The functions $U(x)$ (and thus S_{sub} and S_{tot}), $U_p(p)$, $\lambda(x)$ and $f_1(p)$ are recalculated at every step, through Eq. 35, Eq. 32, Eq. 34 and Eq. 27 respectively. In this way the value of $\xi(x = 0)$ obtained by integration of $f_1(p)$ through Eq. 29 in general does not match the value of ξ_1 given by Eq. 36, hence we restart the procedure with a different ratio R_{sub}/R_{tot} until this necessary condition is satisfied. Having thus found a solution, one is able to calculate the acceleration time which corresponds to it according to Eq. 31. Adjusting p_{max} obviously allows us to find the whole self-consistent solution for a fixed age of the SNR.

4 STREAMING INSTABILITY

Magnetic fields can be generated by streaming instability induced by cosmic rays, although alternative models have also been proposed (see e.g. Giacalone & Jokipii 2007). Streaming instability can be induced in a resonant (Bell 1978) or non-resonant (Bell 2004) way, depending on the type of interaction between particles and waves. In the former case, the unstable modes are Alfvén waves, while in the latter case the modes are almost purely growing modes and do not correspond to Alfvén waves. A satisfactory description of the interaction of particles with these waves is missing, therefore it is very problematic at the present time to describe cosmic ray diffusion in a background of waves which are excited non-resonantly (see Zirakashvili & Ptuskin 2008a; Reville et al. 2008, for some recent attempts). Moreover, even the jump conditions at the subshock and in the precursor would be different from the ones typically adopted and it is not clear as yet which form the wave terms should have. In the context of SNRs, Amato & Blasi (2009) showed that the non-resonant modes are bound to be relevant only in the early stages of the evolution, while for most of the history of the SNR, streaming instability should be dominated by the excitation of resonant waves. For these reasons we chose to focus only on the self-generation of resonant Alfvén waves, leaving other cases, including the phenomenological approach of Bell & Lucek (2001), for future work.

The stationary equation for the growth and transport of magnetic turbulence reads (e.g. McKenzie & Völk 1982):

$$\frac{\partial \mathcal{F}_w(k, x)}{\partial x} = u(x) \frac{\partial \mathcal{P}_w(k, x)}{\partial x} + \sigma(k, x) \mathcal{P}_w(k, x) - \Gamma(k, x) \mathcal{P}_w(k, x), \quad (38)$$

where \mathcal{F}_w and \mathcal{P}_w are, respectively, the energy flux and pressure per unit logarithmic bandwidth of waves with wavenumber k , $\sigma(k, x)$ is the rate at which the energy in magnetic turbulence grows and $\Gamma(k, x)$ is the rate at which it is damped. This equation is very general, but in the case of modified shocks one should keep in mind the fact that Fourier analysis in k -space is only accurate for wavenumbers such that $1/k$ remains appreciably smaller than the typical length scale of the precursor.

In the following we only consider resonant scattering between the accelerated particles and the magnetic turbulence, which gives for the growth-rate of energy in Alfvén waves:

$$\sigma(k, x) = \frac{4\pi}{3} \frac{v_A(x)}{\mathcal{P}_w(k, x)} \left[p^4 v(p) \frac{\partial f(x, p)}{\partial x} \right]_{p=\bar{p}(k)}, \quad (39)$$

where $\bar{p}(k) = eB/km_p c$ is the resonance condition (see e.g. Amato & Blasi 2006, for a derivation of this expression).

Assuming no damping for the moment, and integrating Eq. 38 in k -space we obtain:

$$\frac{dF_w(x)}{dx} = u(x) \frac{dp_w(x)}{dx} + v_A \frac{dp_c(x)}{dx}. \quad (40)$$

We further assume that only waves of one sign of helicity are generated upstream and that $v_A \ll u$, so that $F_w(x) \simeq 3u(x)p_w(x)$. With these assumptions Eq. 40 becomes:

$$2U(x) \frac{d\alpha(x)}{dx} = V_A(x) \frac{d\xi(x)}{dx} - 3\alpha(x) \frac{dU(x)}{dx}. \quad (41)$$

Provided the cosmic ray generation is efficient and that both M_0 and M_A are much larger than 1, we can neglect the plasma and the magnetic pressure with respect to the kinetic and cosmic ray terms in Eq. 35, hence $\xi(x) \simeq 1 - U(x)$ and Eq. 41 can be solved analytically, returning

$$\alpha(x) = U(x)^{-3/2} \left[\alpha_0 + \frac{1 - U(x)^2}{4M_{A0}} \right]. \quad (42)$$

We could use Eq. 41 directly in our calculations, but it is easy to check that the assumptions which lead to Eq. 42 are well satisfied in all cases of interest. The important physical information to retain at this level is the enhancement of the magnetic field due to adiabatic compression, which clearly shows in Eq. 42 through the factor $U^{-3/2}$.

In the following we consider the case $\alpha_0 = 0$, assuming that all the turbulence is generated via streaming instability, thus

$$\alpha(x) = \frac{1 - U(x)^2}{4M_A(x)U(x)}, \quad (43)$$

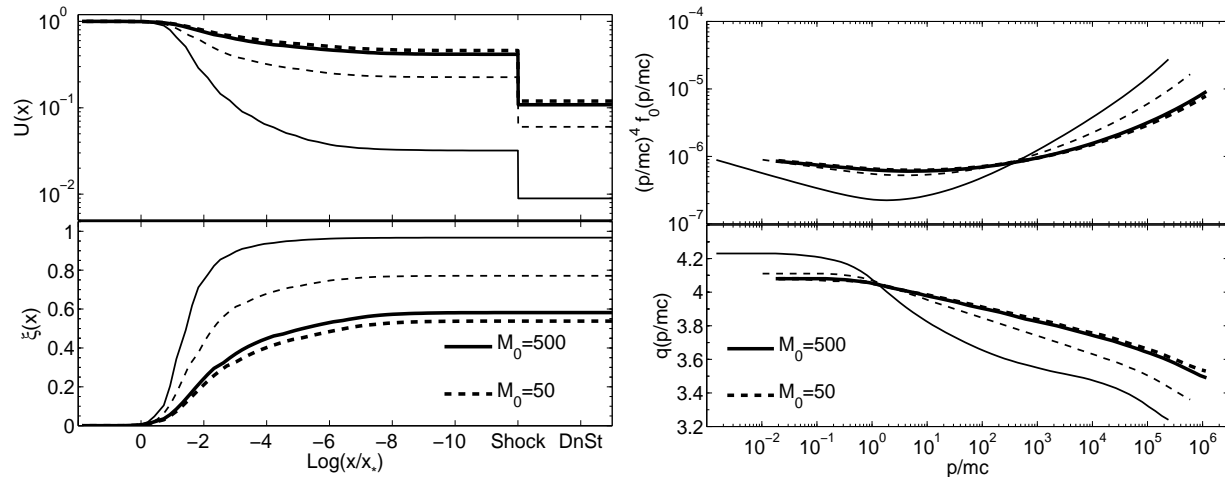


Figure 1. Left panel: velocity (*top*) and cosmic ray pressure (*bottom*) profiles. Right panel: momentum distribution (*top*) and spectral slope (*bottom*) of the cosmic ray spectrum at the shock location. The *thick lines* represent the solution found when the magnetization is correctly treated in the jump conditions, while the *thin lines* show the solution found when it is ignored. Different line-types refer to different values of the Mach number M_0 , as specified in the figure. The location upstream is in units of $x_* = -D(p_{max})/u_0$ and is expressed in base-10 logarithm, while the downstream is taken as homogeneous.

where $M_A(x) = M_{A0}\sqrt{U(x)}$ is the local Alfvénic Mach number.

For the sake of clarity, we stress that contributions of order $1/M_A$ are usually negligible, as in the calculation of the reflection and transmission coefficients or in the treatment of the magnetic energy flux. The only exception to this rule is in the conservation equations for momentum and energy, where the magnetic terms, of order $1/M_A$, are comparable to the ones pertaining the gas, usually of order $1/M_0^2$. Both these contributions are very relevant to the shock dynamics because they affect the compressibility of the system. However, they are extremely small compared to the kinetic and cosmic ray energy, so that Eq. 42 is a very good approximation.

5 RESULTS

In this section we show the results obtained through the algorithm described in Sec. 3, and including only resonant amplification of the magnetic field in the precursor, as described in Sec. 4.

Here and in the following, unless specified otherwise, we assume a SNR age of 1000yr, a circumstellar density of $\rho_0 = 0.5m_p/\text{cm}^3$ and a background magnetic field of $B_0 = 5\mu\text{G}$. The injection parameter ψ is kept fixed: $\psi = 3.7$. We also consider two different circumstellar temperatures $T_0 = 10^4$ and 10^6 K, which for $u_0 = 5900\text{km/s}$ correspond to $M_0 = 500$ and $M_0 = 50$ respectively. This is done in order to investigate different scenarios for shock propagation.

In Fig. 1 the velocity and cosmic ray pressure are shown (*top and bottom panels* on the left respectively) for $M_0 = 50$ and 500 (*dashed and solid lines* respectively) with or without inclusion of the magnetic feedback (*thick or thin lines*). The values found for the relevant parameters in the same cases are reported in Tab. 2.

The most striking effect that comes from the correct treatment of the jump conditions for the magnetic field is indeed the reduced shock modification, visible from the precursor profile: the total compression ratio is found to be $R_{tot} \sim 9$ for both values of the Mach number, while the prediction of standard non-linear theory would be $R_{tot} \sim 112$ for $M_0 = 500$ and $R_{tot} \sim 17$ for $M_0 = 50$.

Clearly, the shallower precursor comes from the decrease of the fraction of bulk energy that is converted into cosmic rays, which however remains considerable: ξ_1 is reduced from more than 90% to around 50–60%, for both values of M_0 (Fig. 1, left bottom panel).

The remaining fraction of bulk energy ends up being converted into heating and energy of the turbulent magnetic field. We define the thermal emissivity as

$$\varepsilon(x) \propto \rho(x)^2 T(x)^{3/2} \propto U(x)^{-2} T(x)^{3/2}, \quad (44)$$

and we report, in the last column of Tab. 2, the ratio between its downstream value computed within modified shock theory (ε_2) and the prediction for the same shock in test particle theory (ε_{tp}). The temperature (next to last column in Tab. 2) and thermal emissivity downstream are always much smaller than in the test-particle approximation, since a considerable fraction of the shock ram pressure is now going into particle acceleration and magnetic field amplification. Nevertheless, it is clear that the effect of the magnetized jump conditions is to enhance both quantities (e.g. T_2 is enhanced by a factor > 100 for $T_0 = 10^4$ K), as a net result of the increased temperature jump T_2/T_1 (Eqs. 11,10) and the reduced compressibility of the upstream plasma (lower R_{tot}).

We should recall that the compression ratios actually felt by cosmic rays depend on the relative velocity between them and the scattering centers, i.e. the Alfvén waves, according to Eq. 22. Nevertheless, when the Alfvén velocity is taken according to Eq. 21, the discrepancy between $S_{sub} - S_{tot}$ and $R_{sub} - R_{tot}$ is usually negligible, the difference in the plasma’s and cosmic rays’ compression ratios being of order 1% at most for the cases in Tab. 2.

As to the magnetic field, the values of B_2 in Tab. 2 are in the correct range inferred from fits to X-ray observations of SNRs, which indicate $B_2 \approx 400 - 500 \mu\text{G}$ for SNRs with u_0 as

Table 2. Comparison of the results obtained with and without inclusion of the magnetic feedback (Λ_B) for the shocks of Fig. 1.

T_0 (K)	Λ_B	ξ_1	$p_{max}(10^6 \text{ GeV}/c)$	R_{sub}	R_{tot}	S_{sub}	S_{tot}	$B_2(\mu\text{G})$	$T_2(10^6 \text{ K})$	$\varepsilon_2/\varepsilon_{tp}$
10^4	No	0.97	0.24	3.58	112.1	3.43	108.7	645.8	0.88	0.030
10^4	Yes	0.58	1.17	3.84	9.22	3.79	9.12	463.9	126.5	0.346
10^6	No	0.77	0.59	3.76	16.6	3.70	16.4	235.0	42.3	0.216
10^6	Yes	0.54	1.14	3.84	8.44	3.79	8.36	425.1	154.8	0.391

high as 5000 – 6000 km/s. This shows that amplification due to streaming cosmic rays can actually account for such high magnetization levels.

Let us now consider the spectrum of the accelerated particles for the same situations discussed above. These are plotted in the right top panel of Fig. 1 (distribution function in momentum space, multiplied by p^4), while the right lower panel shows the spectral slope $q(p)$ at the shock location. For diffusion coefficients that increase with particle momentum, higher energy particles stream further upstream of the shock than the less energetic ones. In the case of a modified shock, this causes the former to experience compression factors even larger than 4, the typical limit for strong shocks in the test particle regime. Since the spectral slope is basically determined by this effective compression ratio, the resulting spectra are concave, softer at the lowest energies and harder at the highest energies. It is intuitively clear that a smoothening of the precursor (i.e. a reduction of the ratio R_{tot}/R_{sub}) results in a reduced concavity of the spectra as compared to the standard prediction of nonlinear models for a given Mach number of the shock. This effect is evident from the comparison of thick and thin curves in Fig. 1, where the dynamical reaction of the amplified field is included: the particle spectrum is roughly close to p^{-4} up to $10^3 \text{ GeV}/c$ and tends to be flatter above this energy, with a slowly changing slope.

Apart from this global concavity of the spectra, three main differences between the case with and without the correct jump conditions are worth being noticed:

1) p_{inj} increases due to the more efficient heating of the downstream plasma, and the fraction of particles injected into the acceleration mechanism (Eq. 25) also slightly increases (from $\eta = 1.03 \times 10^{-4}$ to $\eta = 1.19 \times 10^{-4}$ for $M_0 = 500$). This is consistent with the increase in R_{sub} induced by the magnetic feedback.

2) at the highest energies the spectra are somewhat steeper than usually predicted for strongly modified shocks ($p^{-3.5}$ instead of $p^{-3.2}$), but the total cosmic ray energy is still dominated by the highest momenta;

3) the maximum momentum achieved by non-thermal particles is increased by about a

Table 3. Solution of DSA for different SNR environmental parameters.

T_0 (K)	n_0 (cm $^{-3}$)	B_0 (μG)	ξ_1	p_{max} (10 6 GeV/c)	R_{sub}	R_{tot}	W	B_2 (μG)	T_2 (10 6 K)
10 4	1	5	0.70	1.42	3.71	12.5	141.9	714.2	61.3
10 4	0.5	10	0.54	1.60	3.76	8.18	373.3	579.0	149.8
10 4	0.1	5	0.50	0.74	3.78	7.68	406.8	255.1	173.6
10 4	0.1	1	0.74	0.36	3.74	14.3	89.9	201.9	48.0
10 6	1	5	0.65	1.43	3.73	10.8	1.41	632.2	88.2
10 6	0.5	10	0.51	1.56	3.77	7.75	3.64	546.6	171.5
10 6	0.1	5	0.48	0.72	3.78	7.36	4.00	243.8	192.9
10 6	0.1	1	0.67	0.36	3.78	11.4	0.89	167.8	82.5
10 6	0.01	5	0.12	0.15	3.94	4.50	4.41	54.7	586.0
10 6	0.01	1	0.37	0.16	3.92	6.24	2.17	50.2	302.6

factor 2-5, and reaches a few times 10 6 GeV/c, as a result of the smoothening of the precursor. We point out that during the free expansion phase the shock velocity may be high enough to make the non resonant streaming instability substantially contribute to the magnetic field amplification. This would reflect in a boost of p_{max} for two concurring reasons: first, as a direct consequence of the resulting decrease of the diffusion coefficient; second, as a consequence of an enhanced magnetic feedback that would cause a further reduction of R_{tot} (see Eq. 31 for the time needed to accelerate a particle to p_{max}). For a typical remnant age of $\tau = 1000$ yr, the maximum energies we derive appear to be in qualitative agreement with the knee in the proton spectrum as observed e.g. by KASCADE (Antoni et al. 2005).

Finally, we notice that our results show a weak dependence on the background temperature T_0 (in the range 10 4 – 10 6 K), and therefore on the sonic Mach number, as long as this is much larger than 1. This contrasts with the standard non linear prediction for the increase of the shock modification as $R_{tot} \propto M_0^{3/4}$. The explanation lies in the fact that the magnetic backreaction is much more effective for high M_0 , since it is driven by

$$W = \frac{\alpha_1}{P_1} = \frac{\gamma}{4} \frac{M_0^2}{M_{A0}} \left(\frac{R_{sub}}{R_{tot}} \right)^{\gamma-3/2} \left[1 - \left(\frac{R_{sub}}{R_{tot}} \right)^2 \right] \propto \frac{u_0 B_0}{\sqrt{\rho_0 T_0}}. \quad (45)$$

In Tab. 3 we show the results for several different choices of the environmental parameters: it is clear that the magnetic feedback is never negligible (namely we always have $W \gtrsim 1$) and that the prediction for the compression ratios is consistent with the values inferred from observations. The case with $T_0 = 10^6$ K and low density $n_0 = 0.01$ cm $^{-3}$, which may be representative of the hot phase of the interstellar medium, does not lead to a shock as strongly modified as in the other cases because in such an environment an age older than ~ 1000 yr is needed to achieve a strong modification.

5.1 The effect of the velocity of scattering centers on the spectrum

As discussed in Sec. 3, the propagation of the accelerated particles in the shock region is described by Eq. 23. The terms $u(x) - v_A(x)$ describe the velocity of the scattering centers. Here v_A represents the wave velocity, which, as discussed above, we assume to be the Alfvén speed calculated in the background field B_0 . Provided the waves remain Alfvén waves even in the regime $\delta B/B \gg 1$ this result holds in an exact way. It is however clear that this can only be considered as an ideal case, in that turbulence may change such a picture in a considerable way. One way in which the change may appear is in changing the wave speed. In this section we investigate the effects on the spectrum of accelerated particles and on the shock precursor which derive from calculating the Alfvén speed in the local amplified field, namely:

$$v_A(x) = \frac{\delta B(x)}{\sqrt{4\pi\rho(x)}}. \quad (46)$$

We stress that in our opinion this assumption is totally unjustified from the physical point of view, and we use this case only as a toy model to illustrate how sensitive the results can be to unknown non-linear effects. We notice however that similar approaches have in fact been adopted, for instance by Zirakashvili & Ptuskin (2008b) and also, in some form, by Bell & Lucek (2001).

The net effect of this apparently harmless assumption is that the velocity of the scattering centers is greatly enhanced and this affects in an substantial way the effective compression factor at the subshock and therefore the spectrum, as was already pointed out by Bell (1978) in the context of test particle theory.

In order to illustrate this effect in a quantitative way we run our calculations for $p_{inj} = 3.7p_{th,2}$, $M_0 = 250$, $T_0 = 10^5$ K, $n_0 = 0.5\text{cm}^{-3}$, $B_0 = 5\mu\text{G}$ and an age of the system of $\sim 1000\text{yr}$. The results are illustrated in Fig. 2. The solid lines are obtained using the Alfvén speed calculated in the background magnetic field B_0 , as in the previous section. In this case the precursor is very evident (top left panel in Fig. 2) and the spectrum of accelerated particles is concave (top right panel in Fig. 2).

The dash-dotted lines refer to the case in which the Alfvén speed is calculated using the amplified magnetic field, but this definition is used only in the transport equation, while the rate of growth of unstable waves is left unchanged. As a consequence of the reduced effective compression ratio of the scattering centers' velocity felt by accelerated particles, the spectrum becomes softer, as illustrated in the right panels of Fig. 2. In fact one can

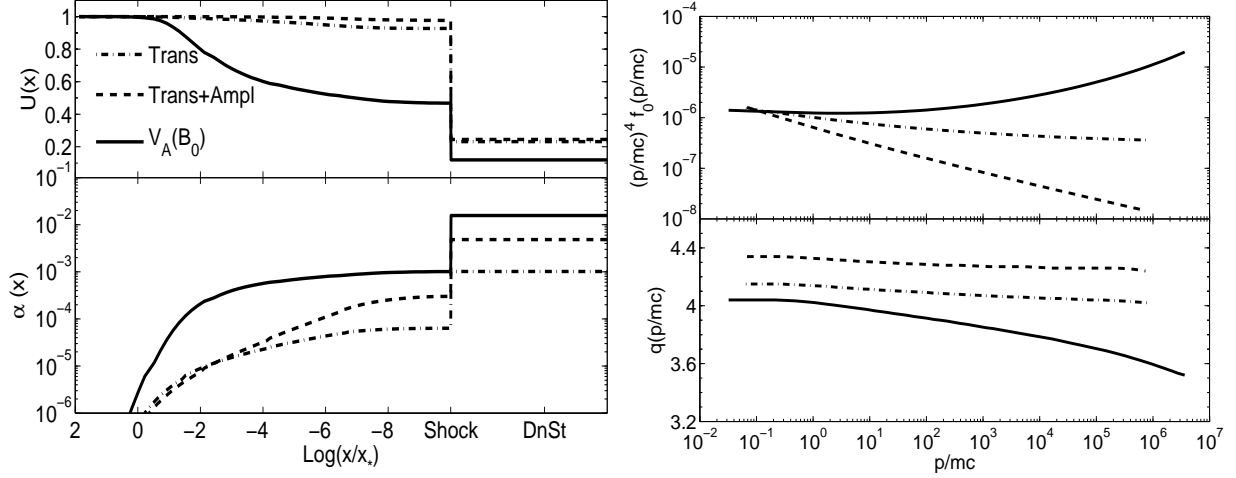


Figure 2. Left panel: velocity (*top*) and normalized magnetic energy density in the shock region (*bottom*). Right panel: momentum distribution (*top*) and spectral slope (*bottom*) of the cosmic ray spectrum at the shock location. The *solid lines* refer to the solution found when the Alfvén speed is calculated in the background field. The *dash-dotted lines* refer to the case in which the modified Alfvén speed is used only in the transport equation, while the *dashed lines* refer to the case in which the modified Alfvén speed is used both in the transport equation and in the growth rate of unstable modes.

see that the spectrum becomes even steeper than p^{-4} at all momenta. When the modified Alfvén speed is used also in the growth rate (dashed lines), the effect is even more dramatic and the spectrum has a slope ~ 4.3 at all momenta and the concavity is hardly visible. Moreover, the amplified field becomes smaller in the latter two cases, which also results in lower maximum energy of the accelerated particles, as visible in the right panels of Fig. 2.

Despite the fact that the assumption of very large v_A could be unphysical, and certainly not well justified from the theoretical point of view, we cannot refrain from being very concerned by the dependence of the results on the value of the effective velocity of the scattering centers. On the other hand, having larger fields does not necessarily imply faster scattering centers. For instance, the non-resonant waves discussed by Bell (2004) and Amato & Blasi (2009) are almost purely growing modes and one could expect that they may be almost stationary in the fluid frame. In addition, one should keep in mind that if the velocity of the waves becomes too high, they may generate shocks in the background plasma and damp their energy on it, so that those waves do not contribute to the scattering of particles.

6 THE (REDUCED) ROLE OF TURBULENT HEATING

Here we address another very important issue, already introduced in the work by Caprioli et al. (2008a). The correct inclusion of the turbulent magnetic field in the calculation of the jump conditions very naturally leads to values of R_{tot} that allow to fit the X-ray observations in a

totally new way in the perspective of cosmic ray modified shocks, i.e. without invoking the presence of additional gas heating mechanisms.

It is well known that the shock dynamics, and in turn the particle acceleration process, is very sensitive to the presence of non-adiabatic gas heating, since a hotter upstream plasma is naturally less compressible. The low compression ratios inferred from observations have been often explained by invoking mechanisms of non-adiabatic heating in the precursor, such as *Alfvén heating* (Völk & McKenzie 1981; McKenzie & Völk 1982) and *acoustic instability* (Drury & Falle 1986; Wagner, Falle & Hartquist 2007).

Acoustic instability develops when sound waves propagate in the pressure gradient induced by cosmic rays in the shock precursor. The instability results in the formation of weak shocks upstream, which cause heating in the precursor, thereby reducing the compressibility of the plasma. Wagner, Falle & Hartquist (2007) showed that there is however a range of steady state solutions characterized by moderate cosmic-ray acceleration and compression ratios significantly larger than 7.

Alfvén heating (also called turbulent heating) is a generic expression which is supposed, at least in principle, to apply to any damping mechanism for Alfvén waves, and may result for instance from ion-neutral damping or non-linear Landau damping, depending on the ionization level of the background plasma. Although the initial mathematical approach of McKenzie & Völk (1982) was based on the assumption of non-linear Landau damping, the formalism was generic enough that it could be adapted to any damping mechanism, and this is indeed what happened. The common formulation of the Alfvén heating assumes that some fraction of the energy in the form of waves is damped into the thermal energy of the background plasma, independently of the details. Notice that this formalism does not distinguish among modes with different wavenumber k , therefore this type of calculations is intrinsically insensitive to the spectrum of turbulence and should not be used to infer information on the shape of the diffusion coefficient. Notice also that whenever applied to the case of non-linear Landau damping, the mechanism is effective only when $u_0 \ll 4000 \text{ km/s} (T_0/5 \times 10^5 \text{ K})^{1/2}$ (Völk & McKenzie 1981). It is not obvious that this condition holds in the young SNRs listed above, since typically $u_0 \sim 3000 - 6000 \text{ km/s}$.

A few other points are worth being mentioned: 1) Alfvén heating was first introduced in order to avoid that the amplified magnetic field could reach the nonlinear regime. On the other hand the same formalism is now used even in situations in which $\delta B \gg B$. Much care should be taken in using these expressions in the nonlinear regime. 2) The formalism

Table 4. Alfvén heating effects

ζ	ξ_1	$p_{max}(10^6 \text{ GeV}/c)$	R_{sub}	R_{tot}	B_1/B_0	W	$B_2(\mu\text{G})$	$T_2(10^6 \text{ K})$	$\varepsilon_2/\varepsilon_{tp}$
0	0.60	1.17	3.76	9.52	25.3	1.941	475.6	114.6	0.317
0.5	0.66	0.84	3.65	10.96	20.8	0.390	379.6	132.6	0.523
0.8	0.65	0.53	3.68	10.76	12.8	0.115	232.5	128.3	0.480
0.99	0.55	0.12	3.85	8.69	2.26	0.005	43.5	162.2	0.553

Solutions found for a shock with $T_0 = 10^6 \text{ K}$, $M_0 = 50$, $\rho_0 = 1m_p/\text{cm}^3$ and $B_0 = 5\mu\text{G}$ when the turbulent heating is taken into account according to Eq. 50, for various $\zeta < 1$ (see Eq. 47). The last column shows the downstream thermal emissivity ε_2 , normalized to the value it would have for the same shock in the test particle regime.

introduced by Völk & McKenzie (1981) and McKenzie & Völk (1982), and later adopted by virtually all authors willing to include Alfvén heating, is based on the implicit assumption that there is a rapid damping of all energy of waves onto the background plasma (as stressed above, this was done in order to avoid excessive magnetic field amplification). This inhibits the generation of magnetic field, so that large magnetic fluctuations become unfeasible. In other words, the standard treatment of Alfvén heating, which corresponds to having $\sigma(x) = \Gamma(x)$ in Eq. 38, is incompatible with having large values of the turbulent field at the shock, as we show below in a quantitative way.

In the following we assume that the damping rate is limited to a fraction of the growth rate, namely that

$$\Gamma(x) = \zeta\sigma(x). \quad (47)$$

An equation describing the Alfvén heating of the precursor can be obtained by taking the derivative of the equation for the conservation of energy together with the equations of transport of waves and cosmic rays (see the derivation of Eq. 9 in McKenzie & Völk 1982). Under the above assumptions we obtain:

$$\frac{\partial}{\partial x} [P_{TH}(x)U(x)^\gamma] = \zeta(\gamma - 1)V_A(x)U(x)^{\gamma-1}\frac{\partial\xi(x)}{\partial x}, \quad (48)$$

and P_{TH} is now the pressure of the plasma including the effects of the turbulent heating. Clearly, in the limit of adiabatic evolution of the precursor, one has $\zeta = 0$ and Eq. 15 is recovered.

It has been shown (Berezhko & Ellison 1999; Amato & Blasi 2006) that a good approximation to the solution of Eq. 48 is

$$P_{TH}(x) \simeq P(x) \left\{ 1 + \zeta(\gamma - 1)\frac{M_0^2}{M_A} [1 - U(x)^\gamma] \right\}. \quad (49)$$

where $P(x)$ is the plasma pressure as calculated taking into account only adiabatic compression in the precursor, Eq. 15. This expression, which serves as an equation of state for the gas in the presence of effective Alfvén heating, reduces to the standard Eq. 50 of

Berezhko & Ellison (1999) for $\zeta = 1$, while, for $\zeta < 1$ the damping of the waves is mitigated and an effective amplification of the magnetic field is allowed.

The change in the equation of state of the gas also manifests itself in the shock dynamics. The new relation between the compression ratios R_{sub} and R_{tot} reads

$$R_{tot}^{\gamma+1} = \frac{M_0^2 R_{sub}^\gamma}{2} \left[\frac{\gamma + 1 - R_{sub}(\gamma - 1)}{(1 + \Lambda_B)(1 + \Lambda_{TH})} \right], \quad (50)$$

where we have introduced

$$\Lambda_{TH} = \zeta(\gamma - 1) \frac{M_0^2}{M_A} \left[1 - \left(\frac{R_{sub}}{R_{tot}} \right)^\gamma \right], \quad (51)$$

with a notation which allows a direct comparison between the effects of magnetic feedback and Alfvén heating.

It is widely known that the inclusion of Alfvén heating has an important impact on the total compression ratio, changing its scaling with the Mach number from $M_0^{3/4}$ to $M_A^{3/8}$ (see e.g. Berezhko & Ellison 1999). However the situation is very different when the correct magnetized jump conditions are taken into account. In Tab. 4, we report, for different values of $\zeta = 0, 0.5, 0.8, 0.99$, the solutions of the problem including the full treatment of growth and damping of Alfvén waves according to the approximate analytical solution of Eq. 38 along with the prescription of Eq. 47, namely

$$\alpha_{TH}(x) = (1 - \zeta) U(x)^{-3/2} \left[\alpha_0 + \frac{1 - U(x)^2}{4M_{A0}} \right]. \quad (52)$$

It is clear that the increasing relevance of Alfvén heating (ζ approaching 1) does not lead, in this approach, to a smoother precursor (larger value of R_{sub}/R_{tot}). In fact, the energy transfer from the waves to the plasma, while heating the plasma and reducing its compressibility, is also accompanied by a decrease of W , the ratio of magnetic/plasma pressure. The latter is the parameter which controls the magnetic feedback, which is then reduced. The net effect is a slight increase of R_{tot} for intermediate values of $\zeta = 0.5 - 0.8$. Only if ζ is very close to 1 the shock is less modified than the adiabatic solution with magnetic backreaction, but even this effect is rather limited, since the decrease of R_{tot} is only about $\sim 10\%$ for the case $\zeta = 0.99$.

The main effect of the inclusion of the Alfvén heating is instead a significant reduction of the magnetic field (more than a factor 10 between $\zeta = 0$ and $\zeta = 0.99$), which also leads to a correspondingly lower p_{max} ($p_{max} \simeq 10^5 \text{ GeV}/c$ for $\zeta = 0.99$).

We notice that the downstream temperature is affected in two different ways by the correct jump conditions and by the turbulent heating: the former provides an increase in

the jump T_2/T_1 proportional to W , while the latter results in a higher T_1 . The interplay between the two effects produces the non-monotonic trend of T_2 and of the thermal emissivity ε_2 (shown in the last columns of Tab. 4). The latter is slightly increased by the presence of non-adiabatic heating, with a value around $0.5\varepsilon_{tp}$, basically for all choices of $\zeta > 0$.

A recent investigation on the role of turbulent heating on the properties of cosmic ray modified shocks has been carried out by Vladimirov, Ellison & Bykov (2008). These authors adopt a different recipe for particle injection into the acceleration process (which is a poorly understood issue in any case), that leads to a dependence of the fraction of injected particles on the temperature immediately upstream of the shock. As a consequence they find that dissipation of magnetic turbulence into heat upstream of the shock boosts the injection by a large factor and modifies the cosmic ray spectrum at low energies, although it does not significantly affect the overall acceleration efficiency and the high energy part of the spectrum unless the the fraction of turbulence energy that is transferred to heat gets close to 100%. If the latter is the case, on the contrary, also Vladimirov, Ellison & Bykov (2008) observe a considerable decrease of both the magnetic field strength and p_{max} , in agreement with our results.

Before concluding this section, we summarize our main conclusions concerning the effects of turbulent heating:

- if turbulent heating is in the form of nonlinear Landau damping, it is suppressed for $u_0 \gg 4000\text{km/s}(T_0/5 \times 10^5 \text{ K})^{1/2}$ (Völk & McKenzie 1981), thus it could be important only if the circumstellar temperature is rather high (for instance because the shock is expanding in the pre-supernova stellar wind, or because of a dominance of the hot coronal gas phase) and the shock has already slowed down significantly;
- if the rate of damping of the waves is too close to that of growth ($\zeta \sim 1$), the magnetic field amplification is heavily suppressed and explaining the large levels of magnetization inferred from X-ray observations becomes very challenging;
- if the damping is effective but still allowing sufficient magnetic field amplification, the smoothening of the precursor is roughly unchanged with respect to the results obtained in the case of magnetic feedback alone.

7 CONCLUSIONS

The possibility that the narrow filaments detected in X-rays may result from severe synchrotron losses of high energy electrons has sparked much attention on the issue of magnetic field amplification at shock fronts, since the inferred fields are ~ 100 times larger than the typical interstellar ones. The importance of such magnetic fields for the origin of cosmic rays is immediate: if the turbulent field leads to Bohm diffusion, it is easy to show that particle acceleration at shock fronts may lead to the generation of protons with energy close to the knee.

In a previous paper (Caprioli et al. 2008a) we investigated the hydrodynamical consequences of having a large magnetic field amplified by cosmic ray streaming upstream of the shock and we demonstrated that when the magnetic pressure equals or overcomes the pressure of the background plasma, a condition easy to realize, the compressibility of the plasma is reduced, so to decrease the compression factor of the cosmic ray modified shock and smoothen its precursor.

In the present paper we went beyond the hydrodynamical picture and we solved the combined system of conservation equations, cosmic ray diffusion-convection equation, and equation for the magnetic field amplification in order to investigate the effects of the precursor smoothening on the spectrum of accelerated particles. We find that resonant streaming instability is sufficient to amplify a parallel pre-existing magnetic field to levels which compare well with X-ray observations, if the latter are interpreted as a result of strong synchrotron losses.

In these circumstances, we also confirm the crucial role of the dynamical feedback of the magnetic field, which leads to total compression factors around $\sim 7 - 10$ (to be compared with the typical predictions of standard NLDSA, which gives $R_{tot} \sim 20 - 100$), in good agreement with the values suggested by observations and by fits to the multifrequency spectrum of several SNRs (Völk, Berezhko & Ksenofontov 2005; Warren et al 2005). The reduced compression in the precursor does not inhibit particle acceleration, in fact a fraction 50-60% of the ram pressure at the shock is converted into accelerated particles, for parameters typical of a SNR in the Sedov phase.

The kinetic calculation of particle acceleration in the nonlinear regime was carried out using the approach of Amato & Blasi (2005, 2006). The effects of resonant streaming instability were treated as in Skilling (1975b); Bell (1978), but including the presence of the precursor

which implies magnetic field compression in addition to amplification. We neglected here the possibility of having non-resonant amplification (Bell 2004), for several reasons: 1) the relation between pressure and energy flux of these waves is not yet well defined; 2) the diffusion properties of charged particles in turbulence at wavelengths much shorter than the Larmor radius of particles are not known as yet, and need a dedicated effort of investigation; 3) as showed by Amato & Blasi (2009), the non resonant modes are likely to be especially important in the free expansion phase of a SNR and in the early stages of the Sedov phase.

Perhaps not surprisingly, we find that the effect of magnetic feedback on the spectrum of accelerated particles is that of reducing the concavity. This result can be easily explained based on the reduced compressibility in the shock precursor, caused by the magnetic pressure. The slope of the spectrum typically remains close to 4 at energies smaller than ~ 1 TeV, while being flatter at higher energies (Fig. 1). We also discuss the effect of the amplified field on the velocity of the scattering centers, which causes a further steepening of the spectrum, leading it to get closer to a power law. We find that the spectrum of accelerated particles becomes even steeper than p^{-4} if the velocity of the scattering centers is assumed to be the Alfvén speed in the amplified field. Although this maybe an unphysical assumption, it serves the scope of showing that the results of calculations carried out in this strongly nonlinear regime may well be affected by details which at the present time we are unable to control.

The smoother precursor induced by the magnetic feedback also implies a larger value of the maximum achievable momentum for the accelerated particles. We recall that, as showed by Blasi, Amato & Caprioli (2007), a strong precursor leads to lower the maximum momentum since particles feel a smaller mean fluid speed and therefore a longer acceleration time. Using the acceleration time for modified shocks as calculated by Blasi, Amato & Caprioli (2007), we find that for a 1000yr old remnant the maximum energy can be as high as 10^6 GeV, a factor 5-10 larger than one could find without including the magnetic backreaction. In principle even larger maximum energies can be achieved by taking into account the non-resonant streaming instability in the early phases of the SNR evolution.

Our entire analysis was carried out for the case of a shock propagating in the direction parallel to the ambient magnetic field. The amplification of the background field due to cosmic ray streaming soon changes this situation, in that the magnetic field upstream becomes highly turbulent. Changing the field obliquity should not change much the picture discussed in the present paper as long as the projection of the shock velocity along the field lines remains supersonic. If the supernova shock propagates in a medium in which the

background field is mainly oriented in a given direction, there will be locations at which the shock is quasi-parallel and others at which it is quasi-perpendicular. The implications for the morphology of the non-thermal emission require a dedicated investigation, in that the quasi-perpendicular regions could be expected to be bright because of the more efficient perpendicular configuration, but quasi-parallel regions could be expected to be bright because of more efficient magnetic field amplification. Clearly the degree to which these expectations may be true depends on many details which at present are not under control, although observations of specific astrophysical objects (e.g. SN 1006) will provide us with important information on this issue.

Finally, we investigated the effect of turbulent heating on the shock modification. We generalized the formalism introduced by Völk & McKenzie (1981) and McKenzie & Völk (1982) in order to account for the possibility that a finite fraction of the wave energy is damped on the background plasma. We find that, although in general the heating of the upstream plasma results in a decrease of the compressibility, the effect is at most competitive, if not subdominant, compared with the dynamical feedback of the amplified magnetic field on the plasma. When turbulent heating is indeed dominant, a fraction very close to unity of the wave energy is transformed in thermal energy, and the main effect is that of suppressing the growth of the magnetic field, so that it becomes challenging to explain X-ray observations in terms of severe synchrotron losses. Moreover in this case the maximum energy of accelerated particles is drastically reduced and it becomes much smaller than the knee.

It is worth stressing that while the magnetic feedback on the background plasma is based on well known physics, the turbulent heating is treated at best in a very phenomenological way, with little attention to the specific physical processes that may be responsible of the non-adiabatic heating and on how modes with different wavelengths are damped. In these circumstances we think that the effect of magnetic feedback is much more solidly assessed and should be considered as the chief mechanism for reducing the compression in cosmic ray modified shock waves with evidences for amplified magnetic fields.

On the other hand, the role of turbulent heating may become much more important for older remnants, when the shock velocity drops below $\sim 2000\text{km/s}$. At these times, the amplification of the magnetic field may have become less important from the dynamical point of view, so that the magnetic feedback is also less important. These stages, as discussed by Ptuskin & Zirakashvili (2005) play an important role in determining the spectrum of cosmic rays observed at the Earth, at least at energies much lower than the knee energy.

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